Lecture 23

- Given 3 doors : 1 car , 2 goats
- you pick one door , Monty opens another door to show a goat.
- So now you can stay at the door you picked or switch.
- you should switch.

↳ This is related to conditional probability.

↳ $P(E|F) = \frac{P(E \cap F)}{P(F)}$

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- What happens if E and F are completely independent?

  - $P(E|F) = P(E)$
  - Then $P(E) \cdot P(F) = P(E \cap F)$

  * This is how we prove E and F are independent!! *

$$P\left[\text{car in } A \mid \text{chose } A\text{, then host opens } B\right]$$

$$= \frac{P\left[\text{car in } A \cap \text{chose } A\text{, then host opens } B\right]}{P\left[\text{chose } A\text{, then host opens } B\right]}$$

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)}{\left[\left(\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 0\right) + \left(\frac{1}{3} \times 1\right)\right]} = \frac{1}{3}$$

Bayes Theorem:

$$P(F|x) = \frac{P(x|F) \cdot P(F)}{P(x|F)P(F) + P(x|\bar{F})P(\bar{F})} = \frac{P(x|F) \cdot P(F)}{P(x)}$$